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The tenth proof was given in 1844 by Donkin in *An essay on the Theory of the Combination of Observations* printed at Oxford, and in 1855 translated in Liouville's *Jour. Math.,* Vol. XV, pp. 297—322. The usual rules for the adjustment of observations are here deduced by a kind of metaphysical statics from the laws of Mechanics. No law of facility is employed.

The eleventh proof was given in 1850 by John Herschel in a review, *Quetelet on Probabilites,* in the *Edinburgh Review,* Vol. XCII, pp. 1—57. It deduces the exponential law of facility by reasoning which is not received by the best mathematicians. See Glaisher's paper quoted above, and Ellis in *Lond. Phil. Mag.,* 1850, Vol. XXXVII, pp. 321—328.

A twelvth proof of the law of facility of error "resting on a natural and obvious assumption" was given by Donkin in the *Quart. Jour. Math.,* 1857, Vol. I, pp. 152—162. The assumption does not appear however to be perfectly obvious.

A thirteenth investigation which may be mentioned is by Crofton, *On the Proof of the Law of Errors of Observations* in the *Phil. Trans.... * London for 1870, pp. 175—188, in which the law of facility of error is investigated, on the hypothesis that an error arises from the joint operation of a large number of small errors, positive and negative errors not being equally probable *a priori.* The discussion is not very clear.

Although no one of these thirteen proofs is perfectly satisfactory, yet each is valuable as setting the subject in a new light and illustrating the laws of thought which have led to the universal employment of the arithmetical mean, and other rules for the adjustment of observations. No discussion of their relative values can be attempted here. The critical papers of Ellis and of Glaisher, quoted above, may in this connection be mentioned as of great value to students of the theory of the subject.

ON THE ROTATION OF SATURN.

BY PROF. A. HALL.

On the night of December 7th while observing Japetus I noticed a bright spot on the ball of Saturn. The spot was 2″ or 3″ in diameter, round and well defined, and of a brilliant white color. It was north of the Ring, nearly midway of the disk in the direction of a circle of declination, and when first seen was about one third the distance from the center of the Ball toward the following edge. The transit of the spot over the center of the disk or rather over the line bisecting the disk and perpendicular to the major axis of the Ring, occurred at 6^h 18^m, Washington m. t. On the next day letters were sent by Admiral Davis to astronomers in different parts of the country asking them to assist in observing the appearance of the spot. With these letters was sent an ephemeris of the spot, computed by assuming the time of the rotation of Sutnrn to be

10^h 29^m $16^s.8$ m. t.

which is given in nearly all the modern text books as Sir W. Herschel's last and most accurate determination. Although this time of rotation proved to be erroneous, and thus the ephemeris was, perhaps, worse than useless, fortunately the spot was observed on Dec. 10th by Professor Maria Mitchell, at the Vassar College Observatory, Poughkeepsie, New York; by Mr. Lewis Boss at the Dudley Observatory, Albany; by Mr. D. W. Edgecomb at Hartford, Connecticut, and by Messrs Alvin Clark and Sons at Cambridgeport, Massachusetts. The spot was observed at Washington until Jan. 2, when having become faint and indistinct, and the weather being bad and the position of the planet unfavorable, the observations were given up. The following table gives the dates and the Washington mean time when the spot was observed to be central on the disk of the planet, the place of observation, the observer and his remarks, and the weight of the observation.

1876, W. m. t.	Place	Observer	Wt.	Remarks
Dec. 7 6^h18^m	Washington	A. Hall	1.00	
" 10 6 6	Cambridgeport	A. G. Clark	0.50	
" 10 6 3	Hartford	D.W. Edgec'mb	0.50	The spot is oval: its br'tness is twice that of the equat'l belt.
" 10 6 4	Poughkeepsie	M. Mitchell	0.50	The spot veyr bri't at its center; elliptical in form.
" 10	Albany	L. Boss		Spot not seen certainly till past cent.
" 13 5 47	Washington	A. Hall	1.00	
" 13 5 50	"	J. R. Eastman	0.50	
" 13 5 38	Cambridgeport	A. G. Clark	0.50	
" 16 5 31	Washington	A. Hall	0.50	Ext'mely poor im.
" 16 5 6	Cambridgeport	A. G. Clark	0.25	Very bad seeing.
" 19 5 2	Washington	J. R. Eastman	0.50	
" 19 5 6	"	A. Hall	1.00	
" 19 5 7	"	S. Newcomb	1.00	
" 19 4 48	Cambridgeport	A. G. Clark	0.25	Very bad seeing.
" 21 8 19	Washington	A. Hall	1.00	
" 27 7 35	"	"	1.00	
" 27 7 42	"	J. R. Eastman	0.50	
" 30 7 25	"	A. Hall	1.00	
Jan. 2 7 9	"	"	1.00	

The observations at Washington with the large refractor were made by placing the wire of the micrometer perpendicular to the major axis of the Ring, and, with the aid of the driving clock, bisecting the disk of the planet with the wire; and in this way estimating the time when the spot came to the center of the disk. Professor Eastman observed with the 9½ inch refractor. The observation by Professor Newcomb, Dec. 19, was made with the large refractor, and we endeavored to be independent in our determination of the time. In assigning the weights I have given to an observation with the large refractor the weight unity, and to all the others a weight of one half, except in a few cases where the notes require the weights to be diminished. Toward the latter part of the observstions the spot had become elongated on the preceding side into a bright belt, and for this reason the observations made during this time are not so good as those made earlier, but I have not changed the weights on this account. In the following table the times have been corrected for the aberration of light, and for convenience the different dates are designated by the letters *a*, *b*, *c*, *d* &c. The weights are given in the column *p*.

		Wash. m. t.		*p*.
1876, Dec. 7	*a*.	4^h	55.7^m	1.00
" 10	*b*.	4	41.6	1.50
" 13	*c*.	4	22.5	2.00
" 16	*d*.	3	59.3	0.75
" 19	*e*.	3	40.0	2.75
" 21	*f*.	6	55.0	1.00
" 27	*g*.	6	12.5	1.50
" 30	*h*.	5	59.8	1.00
1877, Jan. 2	*i*.	5	43.6	1.00

In order to deduce from these observations the time of the rotation of Saturn the following method has been used. We have given the position in space of the two right lines, the lines of sight, from the observer to the spot on any two dates and hence the angle between these lines can be computed. It is assumed that the angle lies in the plane passing through the positions of the Earth for the two dates and the position of Saturn for the mean of these dates; also that the vertex of this angle is at the center of Saturn. If we denote by i the inclination of the equator of Saturn to the ecliptic, and by N the longitude of its ascending node: also by i' and N' similar quantities for the plane of the lines of sight, the inclination J of the plane of the equator of Saturn to the plane of the lines will be given by the equation

$$\cos J = \cos i \cos i' - \sin i \sin i' \cos (N - N')$$

where $i = 28^\circ\ 10'$, and $N = 167^\circ\ 53'$.

If θ be the angle between the two lines for any two dates, and φ this angle reduced to the equator of Saturn we shall have

$$\tan \varphi = \tan \theta \cos J.$$

If we denote by $\alpha_1, \beta_1, \gamma_1$; $\alpha_2, \beta_2, \gamma_2$ the coordinates of the Earth for the first and second dates; and by $\alpha_3, \beta_3, \gamma_3$, the coordinates of Saturn for the mean of the dates: and if we represent the equation of the plane passing through these points by

$$Ax + By + Cz + D = 0$$

we shall have

$$A = (\beta_2 - \beta_1)\gamma_3 : \qquad B = (\alpha_1 - \alpha_2)\gamma_3$$

$$C = \alpha_1\beta_2 - \alpha_2\beta_1 + \alpha_2\beta_3 - \alpha_3\beta_2 + \alpha_3\beta_1 - \alpha_1\beta_3 :$$

and

$$\sin i' = \frac{\sqrt{(A^2+B^2)}}{\sqrt{(A^2+B^2+C^2)}} : \qquad \tan N' = -\frac{A}{B}.$$

As the observations show that the time of rotation is nearly $10^h\ 15^m$, if we express φ in minutes of arc the correction to the observed interval, in minutes of time, will be

$$\Delta t = \frac{10.25}{360} \cdot \varphi' = (8.4544)\varphi'.$$

By this method the observed times have been corrected for the motions of the Earth and Saturn, and made comparable with the observation of December 7th.

I now assume Dec. 7.0 as the epoch, and that m is the time from this epoch to the first observation: then if t_n be any observed time, n the number of rotations of Saturn after the epoch and r the time of one rotation we have

$$nr + n = t_n$$

Putting

$$r = r_0 + x : \qquad m = m_0 + y;$$

$$nr_0 + m_0 - t_n = \Delta,$$

where r_0 and m_0 are approximate values, the equation of condition is

$$\sqrt{p}.(nx + y + \Delta) = 0.$$

The assumed approximate values are

$$r_0 = 614^m.5 : \qquad m_0 = 296^m.0$$

The following table gives the angle θ, in minutes of arc, for each of the positions with respect to the first; the values of i' and N', the correction Δt, to the observed time, n the number of rotations of Saturn from the position a, the residual Δ and its weight, and in the last column the values of the residuals found by substituting the resulting values of x and y in the equations of condition.

Position	θ	i'	N'	Δt	n	Δ	p	$v\sqrt{p}$
a	0.00′	—	—	-0.00^{m}	0	$+0.30^{\mathrm{m}}$	1.00	$+2.18^{\mathrm{m}}$
b	9.55	6°02′	349°07′	0.25	7	-3.85	1.50	-3.31
c	22.32	5 43	350 05	0.59	14	-2.91	2.00	-3.51
d	33.60	5 24	351 15	0.88	21	$+2.08$	0.75	$+1.56$
e	46.77	5 0	352 53	1.23	28	$+3.23$	2.75	$+3.68$
f	55.40	4 50	353 18	1.45	33	$+0.95$	1.00	-0.59
g	85.35	4 16	356 43	2.22	47	$+7.22$	1.50	$+5.17$
h	101.03	4 2	358 17	2.62	54	$+1.82$	1.00	-1.90
i	115.90	3 51	359 45	-3.00	61	-0.10	1.00	-4.50

Hence we have the following equations of condition:

$$
\begin{aligned}
0.00\,x + 1.00\,y + 0.30 &= 0\\
8.57\,x + 1.22\,y - 4.72 &= 0\\
19.80\,x + 1.41\,y - 4.12 &= 0\\
18.17\,x + 0.87\,y + 1.80 &= 0\\
46.44\,x + 1.66\,y + 5.36 &= 0\\
33.00\,x + 1.00\,y + 0.95 &= 0\\
57.56\,x + 1.22\,y + 8.84 &= 0\\
54.00\,x + 1.00\,y + 1.82 &= 0\\
61.00\,x + 1.00\,y - 0.10 &= 0
\end{aligned}
$$

The solution of these equations by the method of least squares gives the following values of x and y.

The normal equations being

$$
\begin{aligned}
+\ 13991.43\,x + 349.50\,y + 791.95 &= 0\\
+\ 349.50\,x + 12.50\,y + 12.68 &= 0
\end{aligned}
$$

we have

$$x = -\,0^{\mathrm{m}}.1037 \pm 0^{\mathrm{m}}.0383; \quad y = +\,1^{\mathrm{m}}.88 \pm 1^{\mathrm{m}}.282.$$

The probable error of an observation of weight unity is $\pm\ 2^{m}.49$. Hence the mean time in which Saturn rotates is

$$10^{h}\quad 14^{m}\quad 23^{s}.8 \pm 2^{s}.30$$

This value has been found by assuming that the spot had no proper motion on the surface of the planet; whether this is really the case or not we cannot determine from the observations.

The only determination of this rotation, so far as I know, is the one by Sir William Herschel, Philosophical Transactions, London, 1794, page 48. By observing the different appearances of a quintuple belt from Nov. 11th 1793, until Jan. 16th, 1794. Herschel found the time of rotation to be, in mean time,

$$10^{h}\quad 16^{m}\quad 0^{s}.4\,;$$

and concludes that this time cannot be in error by so much as two minutes. At the time of the discovery of the white spot this determination of Herschel was not known to me; and as stated before, the time of rotation was assumed to be

$$10^{h}\quad 29^{m}\quad 16^{s}.8\,;$$

and through this mistake several observers failed to see the spot. This last time is given by nearly all the astronomical writers, and by several, Proctor, Klein, Wolf &c., it is said to be Herschel's final determination; "schliesslich zu 10 Stunden 29 Minuten 17 Secunden be stimmte." But I have not found that Sir Willim Herschel ever gave this value. As to its origin I cannot speak with certainty; but it may have been found in the following manner. In his Système du Monde Laplace says that Saturn rotates in 0.428; and that the Ring rotates in 0.437, these numbers being expressed in decimals of a day. If we reduce them to hours, minutes and seconds we have

$$0^{d}.428 = 10^{h}\quad 16^{m}\quad 17^{s}.2 \text{ and } 0^{\delta}.437 = 10^{h}\quad 29^{m}\quad 16^{s}.8$$

The first value given by Laplace is Herschel's, correct to the last decimal of a day and is given in some of the older books. The other value it will be seen is the one commonly given. Hence I think it probable that some one, about fifty years ago, took the wrong number from Laplace and reduced it to hours, minutes and seconds, and that the book makers have faithfully copied this mistake. It should be stated however that one American writer, Professor John Brocklesby, gives Herschel's value very nearly correct, viz., $10^{h}\ 16^{m}\ 4^{s}$.

One other mistake on this question may be noticed. In the ninth edition of Sir John Herschel's "Outlines of Astronomy," p. 343, the time of the rotation of Saturn is said to be

$$9^h\ \ 57^m\ \ 1^s.91$$

as determined by Mr. Airy. But Mr. Airy did not determine the rotation of Saturn but of Jupiter, and the time given is the *siderial* time of the rotation of Jupiter, which still needs a small correction on account of aberration. 1877, Jan. 20.

CLASSIFICATION OF PLANE CURVES WITH REFERENCE TO INVERSION.

BY PROF. W. W. JOHNSON, ST. JOHN'S COLLEGE, ANNAPOLIS, MD.

1. THE degree of the inverse of a given curve of the nth degree is in general $2n$, (thus the inverse of a conic is in general of the 4th degree,) nevertheless it is possible so to classify curves that a given curve and its inverse with reference to any point shall belong to the same family.

The polar coordinates $(r'\ \theta')$ of the point on the inverse curve corresponding to the point $(r,\ \theta)$ of the given curve are $r' = k^2 \div r,\ \theta' = \theta$; where k^2 is the modulus of inversion, and the centre of inversion is the pole. Hence denoting the rectangular coordinates by $(x,\ y)$ and $(x',\ y')$ we derive

$$x = \frac{k^2x'}{r'^2} \quad \text{and} \quad y = \frac{k^2y'}{r'^2}.$$

Substituting these values for x and y in the equation of the given curve we have the equation of the inverse curve, r'^2 being finally replaced by $x'^2+y'^2$.

2. Now the highest terms of the given equation being denoted by u_n, those of the $(n-1)$th degree by u_{n-1} &c.; if u_n contains a power of x^2+y^2 as a factor, let this factor be replaced by the like power of r^2. Let the terms u_{n-1} be treated in the same way, and so on with each group of terms of a degree exceeding $\frac{1}{2}n$. Regarding r^2 for a moment as a factor of only *one dimension*, ascertain the degree of the equation in r^2, x and y. Call this the *circular degree* of the equation, and denote it by m. The complete equation of the mth circular degree may then be written in the annexed tabular form. The terms of like degree are written in the same horizontal line and the degree of the eq'n cannot exceed $2m$. The *circular degree* of all the terms above the central line is m.

$$\left.\begin{array}{c}
Ar^{2m} \\
+(Bx+Cy)r^{2(m-1)} \\
+(Dx^2+Exy+Fy^2)r^{2(m-2)} \\
\cdots\cdots\cdots\cdots\cdots\cdots \\
Mx^m+Nx^{m-1}y+\ \ldots\ldots\ldots\ +Ty^m \\
\cdots\cdots\cdots\cdots\cdots\cdots \\
+\ D'x^2\ +\ E'xy\ +\ F'y^2 \\
+\ B'x\ +\ C'y \\
+A'
\end{array}\right\} = 0$$